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Analog black holes in flowing dielectrics

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Abstract

We show that a flowing dielectric medium with a linear response to an external electric field can be used to generate an analog geometry that has many of the formal properties of a Schwarzschild black hole for light rays, in spite of birefringence. We also discuss the possibility of generating these analog black holes in the laboratory.

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I. INTRODUCTION

In recent years, there has been a growing interest in models that mimic some features of gravitational systems in the laboratory [1]. The actual realization of these models relies on systems that are very different in nature. Among them we can cite ordinary nonviscous fluids, superfluids, flowing and non-flowing dielectrics, nonlinear electromagnetism in vacuum, and Bose-Einstein condensates [2]. The basic feature shared by these systems is that the behaviour of the fluctuations around a background solution is governed by an "effective metric". More precisely, the particles associated to the perturbations follow geodesics not of the background spacetime, but of a Lorentzian geometry, described by the effective metric, which depends on the background solution. It is important to notice that this analogy reproduces exactly the kinematical aspects of general relativity [3], but not the dynamics: there is no analog of Einstein's equations in these models (see however [2,4]).

This analogy has permitted the simulation of several configurations of the gravitational field. Particular emphasis has been placed in analog black holes, because the horizon of these "analog" black holes would emit Hawking radiation with a temperature proportional to the surface gravity. It is the prospect of observing this radiation that motivates the quest for a realization of these analog black holes in the laboratory.

The possibility of generating a black hole for photons in a nonlinear medium was considered in [5], where advantage was taken from the equivalence between nonlinear electromagnetism in vacuum and linear electromagnetism in a nonlinear medium. However, it was shown that it is not possible to generate a Schwarzschild black hole with this system. In this article we would like to present a new static and spherically symmetric analog black hole, generated by a flowing isotropic dielectric that depends on an applied electric field. First, the matter of photon propagation in this kind of medium will be examined in the next section. We anticipate that photons experience birefringence, i.e. there are two effective metrics associated to this medium. A photon "sees" one of these metrics or the other depending on its polarization. Although it may be argued that birefringence spoils the whole effective geometry idea, we shall show in Section III that *photons see one and the same black hole, independently of their polarization.* In Section IV we shall examine in detail the case of a medium with a linear dependence in the external field. The features of the black hole (temperature, effective potential) are also exhibited in this section, and it will be shown that there is a simple relation between the two effective metrics. We close in Section V with some comments regarding the possibility of creating these black holes in the lab.

II. PROPAGATING MODES

In this section we shall study the propagation of photons with different polarizations in a nonlinear medium (see Ref. [6] for details and notation). Let us define first the antisymmetric tensors $F_{\mu\nu}$ and $P_{\mu\nu}$ representing the electromagnetic field. They can be expressed in terms of the strengths (E, H) and the excitations (D, B) of the electric and magnetic fields as

$$F_{\mu\nu} = v_\mu E_\nu - v_\nu E_\mu - \eta_{\mu\nu}{}^{\alpha\beta} v_\alpha B_\beta \tag{1}$$

$$P_{\mu\nu} = v_\mu D_\nu - v_\nu D_\mu - \eta_{\mu\nu}{}^{\alpha\beta} v_\alpha H_\beta \tag{2}$$

where v_μ represents the 4-velocity of an arbitrary observer. The Levi-Civita tensor introduced above is defined in such way that $\eta^{0123} = +1$. Since the electric and magnetic fields are spacelike vectors, we will denote the products as $E^\alpha E_\alpha = -E^2, H^\alpha H_\alpha = -H^2$. We will consider here media with properties determined by the tensors $\epsilon_{\alpha\beta}$ and $\mu_{\alpha\beta}$ which relate the electromagnetic excitations to the field strengths by the generalized constitutive laws,

$$D_\alpha = \epsilon_\alpha{}^\beta(E,H)E_\beta \tag{3}$$
$$B_\alpha = \mu_\alpha{}^\beta(E,H)H_\beta. \tag{4}$$

By taking the discontinuity of the field equations ${}^*F^{\mu\nu}{}_{;\nu} = 0$ and $P^{\mu\nu}{}_{;\nu} = 0$ it can be shown that

$$h^\mu = \frac{1}{\mu(kv)}\eta^{\mu\nu\alpha\beta}k_\nu v_\alpha e_\beta, \tag{5}$$
$$Z^{\mu\beta}e_\beta = 0 \tag{6}$$

where k^μ is the wave propagation vector, the vectors e_μ and h_μ represent the discontinuities of the electric and magnetic fields on the surface of discontinuity, and

$$Z^{\mu\beta} = \left(\epsilon^{\mu\beta} - \frac{1}{E}\frac{\partial\epsilon^{\mu\rho}}{\partial E}E_\rho E^\beta\right)(kv)^2 - \frac{(kv)}{\mu H}\frac{\partial\epsilon^{\mu\rho}}{\partial H} \times \eta^{\tau\nu\alpha\beta}H_\tau E_\rho k_\nu v_\alpha - \frac{1}{\mu}\left[k^\mu k^\beta - (kv)v^\mu k^\beta + (kv)^2\eta^{\mu\beta} - k^2\eta^{\mu\beta}\right]. \tag{7}$$

Non-trivial solutions of equation (6) can be found only for cases in which $\det\left|Z^{\mu\beta}\right| = 0$ (this condition is a generalization of the well-known Fresnel equation [7]).

By considering the special case $\mu = \mu_0 =$ const., and

$$\epsilon^{\mu\beta} = \epsilon(E)(\delta^{\mu\beta} - v^\mu v^\beta), \tag{8}$$

Eqn. (6) reduces to

$$\left\{\left[k^2 - (1-\mu\epsilon)(kv)^2\right]\eta^{\mu\beta} - \mu(kv)^2\frac{1}{E}\frac{\partial\epsilon}{\partial E}E^\mu E^\beta + (kv)v^\mu k^\beta\right\}e_\beta = 0. \tag{9}$$

This equation can be solved by expanding e_ν as a linear combination of the four linearly independent vectors v_ν, E_ν, k_ν and $\eta_{\alpha\beta\mu\nu}v^\alpha E^\beta k^\mu$. That is,

$$e_\nu = \alpha E_\nu + \beta\eta_{\alpha\lambda\mu\nu}v^\alpha E^\lambda k^\mu + \gamma k_\nu + \delta v_\nu. \tag{10}$$

With this assumption, Eqn. (6) reads

$$\begin{aligned}
\alpha\left[k^2 - \left(1 - \mu\frac{\partial(\epsilon E)}{\partial E}\right)(kv)^2\right] - \gamma\left[\mu(kv)^2\frac{1}{E}\frac{\partial\epsilon}{\partial E}E^\alpha k_\alpha\right] &= 0\\
\alpha E^\mu k_\mu + \gamma(1-\mu\epsilon)(kv)^2 + \delta(kv) &= 0\\
\alpha(kv)E^\mu k_\mu + \gamma(kv)k^2 + \delta\left[k^2 + \mu\epsilon(kv)^2\right] &= 0\\
\beta\left[k^2 - (1-\mu\epsilon)(kv)^2\right] &= 0 \quad .
\end{aligned}$$

The solution of this system results in the following dispersion relations:

$$k^2 = (kv)^2\left[1 - \mu\frac{\partial(\epsilon E)}{\partial E}\right] + \frac{1}{\epsilon E}\frac{\partial\epsilon}{\partial E}E^\alpha E^\beta k_\alpha k_\beta \tag{11}$$
$$k^2 = (1-\mu\epsilon)(kv)^2, \tag{12}$$

which are associated with the propagation modes

$$e_\nu^- = \rho\left\{\mu\epsilon(kv)^2E_\nu + E^\alpha k_\alpha[k_\nu - (kv)v_\nu]\right\} \tag{13}$$
$$e_\nu^+ = \sigma\eta_{\alpha\lambda\mu\nu}v^\alpha E^\lambda k^\mu, \tag{14}$$

where ρ and σ are arbitrary constants. The labels "+" and "−" refer to the ordinary and extraordinary rays, respectively. Eqns. (11)and (12) govern the propagation of photons in the medium characterized by $\mu = \mu_0$ =const., and $\epsilon = \epsilon(E)$. They can be rewritten as $g^{\mu\nu}_{\pm}k_\mu k_\nu = 0$, where we have defined the effective geometries

$$g^{\mu\nu}_{(-)} = \eta^{\mu\nu} - \left[1 - \mu_0\frac{\partial(\epsilon E)}{\partial E}\right]v^\mu v^\nu - \frac{1}{\epsilon E}\frac{\partial\epsilon}{\partial E}E^\mu E^\nu, \tag{15}$$
$$g^{\mu\nu}_{(+)} = \eta^{\mu\nu} - (1-\mu_0\epsilon)v^\mu v^\nu. \tag{16}$$

The metric given by Eqn.(15) was derived in [8]. The second metric very much resembles the metric derived by Gordon [9]. The difference is that in the case under consideration, $\epsilon = \epsilon(E)$, while Gordon worked with a constant permeability.

III. THE ANALOG BLACK HOLE

We shall show in this section that an analog black hole can be obtained starting from the effective metrics given by Eqns.(15)-(16). It will be convenient to rewrite at this point the inverse of the effective metric given by Eqn.(15) using a different notation:

$$g^{(-)}_{\mu\nu} = \eta_{\mu\nu} - \frac{v_\mu v_\nu}{c^2}(1-f) + \frac{\xi}{1+\xi}l_\mu l_\nu, \tag{17}$$

where we have defined the quantities

$$f = \frac{1}{c^2\mu_0\epsilon(1+\xi)}, \qquad \xi = \frac{\epsilon' E}{\epsilon}, \qquad l_\mu = \frac{E_\mu}{E},$$

Note that $\epsilon = \epsilon(E)$. We have introduced here the velocity of light c, which was set to 1 before.

Taking a Minkowskian background in spherical coordinates, and

$$v_\mu = (v_0, v_1, 0, 0), \qquad E_\mu = (E_0, E_1, 0, 0), \tag{18}$$

we get for the effective metric described by Eqn.(17),

$$g^{(-)}_{00} = 1 - \frac{v_0^2}{c^2}\,(1-f) + \frac{\xi}{1+\xi}\,l_0^2, \tag{19}$$

$$g^{(-)}_{11} = -1 - \frac{v_1^2}{c^2}\,(1-f) + \frac{\xi}{1+\xi}\,l_1^2, \tag{20}$$

$$g^{(-)}_{01} = -\frac{v_0 v_1}{c^2}\,(1-f) + \frac{\xi}{1+\xi}\,l_0\,l_1, \tag{21}$$

and $g_{22}^{(-)}$ and $g_{33}^{(-)}$ as in Minkowski spacetime.

The vectors v_μ and l_μ satisfy the constraints

$$v_0^2 - v_1^2 = c^2, \tag{22}$$

$$l_0^2 - l_1^2 = -1, \tag{23}$$

$$v_0 l_0 - v_1 l_1 = 0. \tag{24}$$

These equations ensure that the flow is everywhere parallel (or antiparallel) to the electric field. This is required in order to preserve the form of the constitutive relations (Eqns.(3) and (4)) [7].

The system of Eqns. (22)-(24) can be solved in terms of v_1, and the result is

$$v_0^2 = c^2 + v_1^2, \tag{25}$$

$$l_0^2 = \frac{v_1^2}{c^2}, \qquad l_1^2 = \frac{c^2 + v_1^2}{c^2}. \tag{26}$$

Now we can rewrite the metric in terms of v_1. The explicit expression for the metric coefficients is:

$$g_{00}^{(-)} = \frac{1 - (v_1/c)^2 (c^2 \mu_0 \epsilon - 1)}{c^2 \mu_0 (\epsilon + \epsilon' E)}, \tag{27}$$

$$g_{01}^{(-)} = \frac{v_1}{c} \sqrt{1 + (v_1/c)^2}\, \frac{1 - c^2 \mu_0 \epsilon}{c^2 \mu_0 (\epsilon + \epsilon' E)}, \tag{28}$$

$$g_{11}^{(-)} = \frac{(v_1/c)^2 - c^2 \mu_0 \epsilon (1 + (v_1/c)^2)}{c^2 \mu_0 (\epsilon + \epsilon' E)}. \tag{29}$$

From Eqn.(27) it is easily seen that, depending on the function $\epsilon(E)$, this metric has a horizon at $r = r_h$, given by the condition $g_{00}(r_h) = 0$ or, equivalently,

$$\left(\mu_0 \epsilon(E) - \frac{1}{v_1^2} \right) \bigg|_{r_h} = \frac{1}{c^2}. \tag{30}$$

The metric given above resembles the not very well known form of Schwarzschild's solution in Painlevé-Gullstrand coordinates [11,12]:

$$ds^2 = \left(1 - \frac{2GM}{r}\right) dt^2 \pm 2 \sqrt{\frac{2GM}{r}}\, dr\, dt - dr^2 - r^2 d\Omega^2. \tag{31}$$

With the coordinate transformation

$$dt_P = dt_S \mp \frac{\sqrt{2GM/r}}{1 - \frac{2GM}{r}}\, dr, \tag{32}$$

the line element given in Eqn.(31) can be written in Schwarzschild's coordinates. The "+" sign covers the future horizon and the black hole singularity.

The effective metric given by Eqns.(27)-(29) looks like the metric in Eqn.(31). In fact, it can be written in Schwarzschild coordinates, with the coordinate change

$$dt_{PG} = dt_S - \frac{g_{01}(r)}{g_{00}(r)} dr. \tag{33}$$

Using this transformation with the metric coefficients given in Eqns.(27) and (28), we get the expression of $g_{11}^{(-)}$ in Schwarzschild coordinates:

$$g_{11}^{(-)} = - \frac{\epsilon}{(1 - (v_1/c)^2 [c^2 \mu_0 \epsilon - 1])(\epsilon + \epsilon' E)}. \tag{34}$$

Note that $g_{01}^{(-)}$ is zero in the new coordinate system, while $g_{00}^{(-)}$ is still given by Eqn.(27). Consequently, the position of the horizon does not change, and is still given by Eqn.(30).

Working in Painlevé-Gullstrand coordinates, we have shown that the metric for the "−" polarization describes a Schwarzschild black hole if Eqn.(30) has a solution. Afterwards we have rewritten the "−" metric in the more familiar Schwarzschild coordinates. Let us consider now photons with the other polarization. They "see" the metric given by Eqn.(16), whose inverse is given by:

$$g_{\mu\nu}^{(+)} = \eta_{\mu\nu} - \frac{v_\mu}{c} \frac{v_\nu}{c} \left(1 - \frac{1}{c^2 \mu_0 \epsilon(E)} \right). \tag{35}$$

Using this equation and Eqns.(25) and (26) it is straightforward to show that

$$g_{00}^{(+)} = 1 - \left(1 + \frac{v_1^2}{c^2} \right) \left(1 - \frac{1}{c^2 \mu_0 \epsilon} \right), \tag{36}$$

$$g_{01}^{(+)} = - \frac{v_1}{c} \sqrt{1 + (v_1/c)^2} \left(1 - \frac{1}{c^2 \mu_0 \epsilon} \right), \tag{37}$$

$$g_{11}^{(+)} = -1 - \frac{v_1^2}{c^2} \left(1 - \frac{1}{c^2 \mu_0 \epsilon} \right). \tag{38}$$

This metric also corresponds to a Schwarzschild black hole, depending on $\epsilon(E)$. Comparing Eqns.(27) and (36) we see that the horizon of both analog black holes is located at r_h, given by Eqn.(30).

By means of the coordinate change defined by Eqn.(33), we can write this metric in Schwarzschild coordinates. The relevant coefficients are given by

$$g_{00}^{(+)} = \frac{1 + (v_1/c)^2 (1 - c^2 \mu_0 \epsilon)}{c^2 \mu_0 \epsilon}, \tag{39}$$

$$g_{11}^{(+)} = - \frac{1}{1 + (v_1/c)^2 (1 - c^2 \mu_0 \epsilon)}. \tag{40}$$

It is important to stress then that *the horizon is located at r_h given by Eqn.(30) for photons with any polarization.* Moreover, the motion of the photons in both geometries will be qualitatively the same, as we shall show below.

IV. AN EXAMPLE

We have not specified up to now the functions $\epsilon(E)$ and $E(r)$ that determine the dependence of the coefficients of the effective metrics with the coordinate r. From now on we assume a linear $\epsilon(E)$,

$$\epsilon = \epsilon_0(\overline{\chi} + \chi^{(2)} E(r)), \tag{41}$$

with $\overline{\chi} = 1 + \chi^{(1)}$. Maxwell's equations read

$$\left(\sqrt{-\gamma}\, \epsilon(r) F^{\mu\nu}\right)_{\nu} = 0, \tag{42}$$

where γ is the determinant of the flat background metric. Taking into account that

$$(F^{01})^2 = \frac{E^2}{c^2}, \tag{43}$$

we get as a solution of Eqn.(42) for a point source in a flat background in spherical coordinates

$$F^{01} = \frac{-\overline{\chi} \pm \sqrt{\overline{\chi}^2 + 4\chi^{(2)} Q/(\epsilon_0 r)^2}}{2c\chi^{(2)}}. \tag{44}$$

Let us consider a particular combination of parameters: $\chi^{(2)} > 0$, $Q > 0$ and the "+" sign in front of the square root in F^{01}, in such a way that $E > 0$ for all r. To get more manageable expressions for the metric, it is convenient to define the function $\sigma(r)$:

$$E(r) \equiv \frac{\overline{\chi}}{2\chi^{(2)}}\, \sigma(r) \tag{45}$$

where

$$\sigma(r) = -1 + \frac{1}{r}\sqrt{r^2 + q} \tag{46}$$

and

$$q = \frac{4\chi^{(2)} Q}{(\epsilon_0 \overline{\chi})^2}. \tag{47}$$

In terms of σ, the metrics take the form

$$ds^2_{(-)} = \frac{2 - (\frac{v_1}{c})^2\, [\, \overline{\chi}\, (\sigma(r) + 2) - 2]}{2\, \overline{\chi}\, (1 + \sigma(r))}\, d\tau^2 - \frac{2 + \sigma(r)}{[2 - (\frac{v_1}{c})^2\, (\overline{\chi}\, (\sigma(r) + 2) - 2)]\, (1 + \sigma(r))}\, dr^2 - r^2 d\Omega^2 \tag{48}$$

$$ds^2_{(+)} = \frac{2 - (v_1/c)^2\, [\, \overline{\chi}\, (\sigma(r) + 2) - 2]}{\overline{\chi}\, (2 + \sigma(r))}\, d\tau^2 - \frac{2}{2 + (v_1/c)^2\, [2 - \overline{\chi}\, (\sigma(r) + 2)]}\, dr^2 - r^2 d\Omega^2 \tag{49}$$

Notice that the (t, r) sectors of these metrics are related by the following expression:

$$ds^2_{(+)} = \Phi(r)\, ds^2_{(-)} \tag{50}$$

where the conformal factor Φ is given by:

$$\Phi = 2\, \frac{1 + \sigma(r)}{2 + \sigma(r)}$$

We shall study next some features of the effective black hole metrics. First, let us make an approximation in the metric coefficients. Notice that the parameter $\chi^{(1)}$ that appears in $\bar{\chi} = 1 + \chi^{(1)}$ is, for ordinary materials, a very small quantity compared to the unity. This implies that the quantity $\bar{\chi}$ can be approximated to 1 in Eqns. (47)-(49). In what follows, we shall work in this approximation.

We would also like to remark that up to this point, the velocity of the fluid v_1 is completely arbitrary. It can even be a function of the coordinate r. We shall assume in the following that v_1 is a constant. This assumption may seem rather restrictive, but it helps to display the main features of the effective metrics in an easy way.

To study the motion of the photons in these geometries, we can use the effective potential. Standard manipulations (see for instance [13]) show that in the case of a static and spherically symmetric metric, the effective potential is given by

$$V(r) = \varepsilon^2 \left(1 + \frac{1}{g_{00}(r)\, g_{11}(r)}\right) - \frac{L^2}{r^2 g_{11}(r)} \tag{51}$$

where ε is the energy and L the angular momentum of the photon.

In terms of $\sigma(r)$, and of the impact parameter $b^2 = L^2/\varepsilon^2$, the "small" effective potential $v(r) \equiv V(r)/\varepsilon^2$ for the metric Eqn.(49) in Schwarzschild coordinates can be written as follows:

$$v^{(-)}(r) = 1 - \frac{2(1 + \sigma(r))^2}{2 + \sigma(r)} - \frac{b^2}{r^2} \frac{(2 - \left(\frac{v_1}{c}\right)^2 \sigma(r))(1 + \sigma(r))}{2 + \sigma(r)} \tag{52}$$

A short calculation shows that v is a monotonically decreasing function of v_1. Consequently, we shall choose a convenient value of v_1/c for the sake of illustrating the features of the effective potential. We present next some plots of the potential for several values of the relevant parameters.

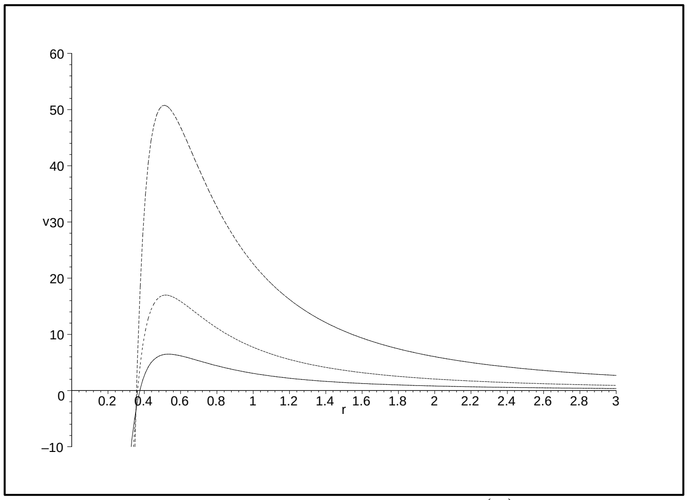

FIG. 1. Plot of the effective potential $v^{(-)}(r)$ for $q = 1$, $b = 1, 3, 5$ (starting from the lowest curve), and $v/c = 0.5$.

The following plot shows the dependence of $v^{(-)}(r)$ with q:

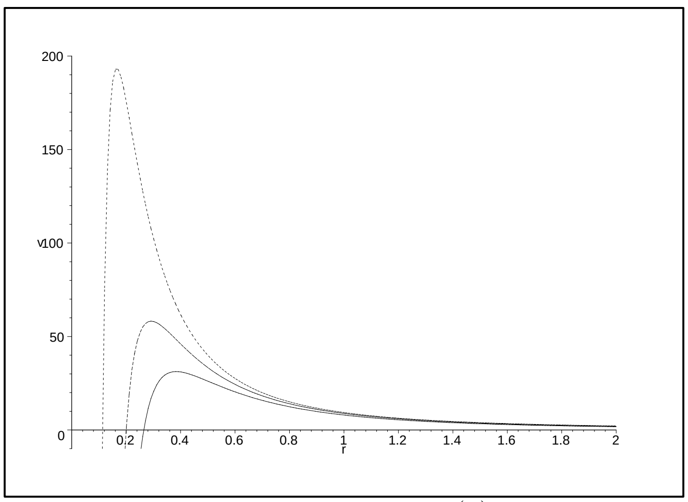

FIG. 2. Plot of the effective potential $v^{(-)}(r)$ for $b = 3$, and $q = 1, 3, 5$ (starting from the lowest curve), and $v/c = 0.5$.

The effective potential for the Gordon-like metric can be obtained in the same way. From Eqns.(51) and (50) we get

$$v^{(+)}(r) = 1 - \frac{2 + \sigma(r)}{2} + \frac{b^2}{2r^2}\,[2 - (v_1/c)^2\sigma(r)] \qquad (53)$$

The following plots show the dependence of $v^{(+)}(r)$ on the different parameters.

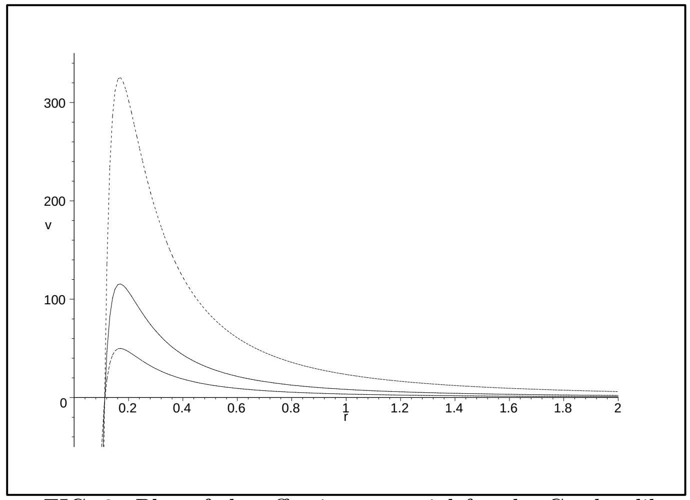

FIG. 3. Plot of the effective potential for the Gordon-like metric, for $q = 1$, $b = 1, 3, 5$ (starting from the lowest curve), and $v/c = 0.5$.

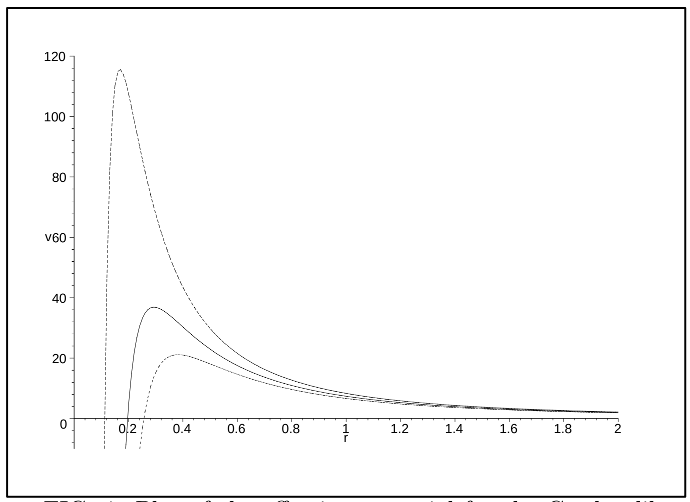

FIG. 4. Plot of the effective potential for the Gordon-like metric, for $b = 3$, $q = 1, 3, 5$.

We see then that, in the case of a constant velocity, the shape of the effective potential for both metrics qualitatively agrees with that for photons moving on the geometry of a Schwarzschild black hole (see for instance Ref. [13], pag. 143).

Let us calculate next the surface gravity of the analog black holes. For a spherically symmetric black hole, it is given by

$$\kappa = \frac{1}{2} \lim_{r \to r_h} \frac{g_{00,r}}{\sqrt{|g_{11}|\; g_{00}}} \qquad (54)$$

In the case of a Schwarzschild black hole, it reduces to the known result, $\kappa = GM/r_h^2$.

For the metrics given by Eqns.(49) and (50) with $\bar{\chi} \approx 1$, the radius of the horizon in terms of the parameter q is:

$$r_h^2 = \frac{(v_1/c)^4\, q}{4\,[1+(v_1/c)^2]}. \tag{55}$$

Using the expressions given above, the result for the surface gravity of the "−" black hole is

$$\kappa^{(-)} = \frac{2}{\sqrt{q}}\frac{1+(v_1/c)^2}{(v_1/c)[(v_1/c)^2+2]} \tag{56}$$

With the surface gravity we can calculate the temperature of the black hole, which is given by

$$T^{(-)} \equiv \frac{\hbar}{2\pi k_B c}\kappa = \frac{\hbar}{\pi k_B c\sqrt{q}}\frac{1+(v_1/c)^2}{(v_1/c)((v_1/c)^2+2)} \tag{57}$$

A short calculation shows that the temperature of the black hole described by the Gordon-like metric coincides with this result. This is not surprising though, because of the conformal relation between the two metrics, given by Eqn.(50) [14].

V. DISCUSSION

We have shown that a flowing inhomogeneous dielectric that depends linearly on an external electric field generates two effective metrics for photons. A particular configuration of the fluid plus the electric field was shown to be an effective black hole, with a radius that depends on the function $\epsilon(E)$. The existence of two metrics reflect the birefringent properties of the medium. Although some claims have been made that birefringent materials spoil from the beginning the idea of an effective metric, we have shown here that for a special configuration, although two metrics are present, photons with different polarizations experience the same horizon, and the temperature associated with the horizon is the same. Moreover, as seen from the plots of the effective potential, the motion of these photons in the medium will depend on their polarization, but is qualitatively the same for both types of photons.

The structures obtained in this article are analogs of gravitational black holes. What can we say about the possibility of generating this black hole in laboratory experiments? From the equation

$$r_h^2 = \frac{(v_1/c)^4\, q}{4\,[1+(v_1/c)^2]}.$$

we see that even if we have some special material and/or a large amount of charge in such a way that $\sqrt{q}$ is large, we still have to cope with the factor $(v_1/c)^2$, which is bound to be very small. The result for r_h is then unacceptably small. Consequently, this black hole cannot be built in the laboratory with present-day technology. In spite of the negative result, the striking similarities between the analog black hole for photons we have presented here with the Schwarzschild black hole suggest that the possibilities offered in this respect by dielectric flowing materials should be explored. Some modifications to the scheme considered here (like geometries with different symmetries and/or more complicated charge distributions) are currently under consideration.

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